

# SIR ISAAC NEWTON'S *DE GRAVITATIONE ET AEQUIPONDIO FLUIDORUM*

translated by

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It is agreed that there are two methods to teach the science of gravitation and of the balance of fluids and solids in fluids. However far it belongs to the science of mathematics, it is plain that it is being exceedingly detached from physical considerations. And here, accordingly, by the singular reason of its propositions from detached [abstracted] principles, and they are sufficient to the attentive, by geometrical manners, I conclude to demonstrate. Then with these doctrines from natural philosophy somehow is the relationship to be estimated, to which extent for the most part the explaining its phenomena in detail is to be adapted, and thus when its use thereafter would be particularly manifest and the certainty of the principles is probably confirmed, I will not regard it a burden yet to illustrate with propositions from abundant experiments: so that, notwithstanding this more open kind of judging being set forth in scholia, is not to be confused with the former that I teach through lemmas, propositions, and corollaries.

The fundamentals from which this science is demonstrated are either called definitions somehow; or axioms and postulates (that) are by no one not conceded. And these I treat forthwith.

## Definitions

The names, quantity, duration. and space, are known for as much as they are able to be defined by other names.

Def. 1: Place is part of space which things fill evenly.

Def. 2: Body is that which fills place.

Def.3: Rest is remaining in the same place.

Def. 4: Motion is change of place.

NOTE: I have said that body fills place; that is that it so fills it that another thing of the same kind or some other body is completely excluded, forasmuch as it is impenetrable. Let it be, however, that place is called the part of space in which things inhere evenly, but so far as here bodies and not penetrable things are being examined, I have chosen to define (that to be) a part of space which something fills.

Moreover, with the body being examined here to be set forth not insofar as it is provided with the physical substance of sensible qualities but only insofar as it is some moveable and impenetrable extension [magnitude]; accordingly, it has not been defined by a philosophical man-

ner, but I detached [abstracted] the sensible qualities (which also the philosophers ought to abstract, unless I err and claim to assign being excited [moved] only to modes of thinking different from the motions of bodies); I have posited only as many of the properties as are required for local motion. Thus far, as if in the place of physical bodies, you can understand the abstract forms in the same manner they are contemplated by geometers when they assign motion to them, as that done in Propositions 4 & 8, Book I of Euclid's *Elements*. And in demonstrating the tenth of the definitions in Book XI, this ought to be done; since indeed it would be barely counted among the definitions and rather ought to be demonstrated among the propositions, unless perhaps it be taken for an axiom.

I have earlier defined motion to be change of place, because those motions, passage, transference, migration, etc., are seen to be synonymous names. But if it be preferred, let motion be passage or transference of a body from place to place.

For the rest in these definitions, when I would have supposed space the distinction from a body is given, and motion relative to a part of space therein, not however relative to the position of a contiguous body have I determined it [lest this be assumed as gratuitously against Cartesians, I shall endeavor to I do away with his fictions].

I can explain his doctrine in the following three propositions: 1. That for each body only one proper motion fits from the truth of things (Article 28, 31, & 32; Part 2, *Principiae Philosophae*), in which transference is defined to be of the one part of matter or one body from the vicinity of those bodies which immediately are co-tangent with it, and inasmuch as it is seen to be at rest, to the vicinity of others. (Article 25, Part 2 & Article 28, Part 3; *Principiae*) 2. That through a body's own motion, according to this definition, transference not only will be understood of any particle of matter, or a body formed of parts being at rest among themselves, but that whole which is simultaneously transferred; albeit reciprocally this itself is able to consist with many parts which themselves may have another motion. (Art 25, Part 2, *Prin.*) 3. That in addition to this motion proper to each body, there can yet be motion actually in it through participating in innumerable others (or to the extent that it is a part of other bodies having a different motion): appertaining (Art. 31, Part 2, *Prin.*): which nevertheless are not motions in the philosophical sense both when speaking rationally (Art. 29, Part 3) and according to the truth of things (Art. 25, Part 2 and Art. 28, Part 3), but only improperly and according to the sense of the vulgar. (Art. 24, 25, 28, & 31; Part 2 and Art. 29. Part 3.) That kind he seems to describe (Art. 24, Part 2 & Art. 28, Part 3) is the action by which some body may migrate from one place to another.

And just as motion consists of two kinds, obviously the proper and the derived, so two kinds of place are assigned from which this motion may be accomplished, which are the surfaces of the immediately surrounding bodies (Art. 15, Part 2), and the situation within other bodies wherever (Art. 13, Part 2 & Art. 29, Part 3).

Now, just how confused and discordant with reason this doctrine is, not only the absurd consequences convince, but also Descartes himself, by contradicting himself, seems to acknowledge. He has said, for instance, (that) the earth and the rest of the planets properly, and speaking according to the philosophical sense, are not moved, and without reason and by speaking only vulgarly, he has said that it is itself moved on account of being transferred relative to the fixed

stars (Art. 26, 27, 28, 29; Part 3). But afterwards, nevertheless, he posits in the earth and the planets an endeavor of receding from the sun as if from the center of a circle that is moved, by which through a like endeavor in the whirling vortices they are balanced in themselves at a distance from the sun. (Art. 140, Part 3). What then? either this endeavor of the planets according to Descartes is truly and philosophically (derived) from rest, or is it rather derived from vulgar and non-philosophic motion? But Descartes seeks further that the comet endeavors less to recede from the sun when it is advanced into the first vortex and the position among the fixed stars entirely remaining, it does not yet obey the pull of the vortices but is transferred relative to it from the vicinity of the contiguous ether, and thus far, philosophically speaking would whirl around the sun; as much as afterwards the comet together with the matter of the vortex has torn away and does so as, according to the philosophical nature of things. And when the whirling of the comets around the sun in his philosophical sense should not effect [produce] an endeavor of receding from the center, which whirling in the vulgar sense can effect [produce], surely he ought to acknowledge motion in the vulgar more than the philosophical sense.

Secondly, he is seen to contradict himself when he posits a unique motion to be suitable for each body according to the truth of things, and yet he determines to consider that motion from our imagination, defining those being transferred from the vicinity of bodies to be not those which are at rest but those which, as if being at rest, are only looked upon as (if) accidentally moving, just as, in Article 29 & 30, Part 2, is explained more broadly. And from this he reckoned himself to be able to avoid the difficulties around the mutual transference of bodies, whereby evidently one more than another is said to be at rest when (its) position between the banks would not be changed. (Art. 15, Part 2). But just as be manifest, supposing that the matter of vortices is only seen by man, if you will, as being at rest and (then) the earth, philosophically speaking, is simultaneously at rest; supposing also that anyone else at the same time looks upon the same matter of vortices as (if) moving circularly and (then) the earth philosophically speaking is not at rest. In the same way a ship on a sea simultaneously will be moved and not moved. And that I am not assuming motion in the wider sense of the vulgar, and by which there are innumerable motions of these bodies, but in his philosophical sense by which he has said there is only one (motion) in body in any place, and it is proper to itself and suitable from the nature of things (not our imaginations).

Thirdly, he is seen by no means to agree with himself when he posits a single motion to be suitable to every body according to the nature of things, and nevertheless (Art. 31, Part 2) innumerable motions to appertain to each single body actually. For motions which actually belong to some body are actually natural motions, and thus motions in the philosophical sense and according to the truth of things, although he insisted (that) it was motion in the vulgar sense alone. Add that, when any whole is moved, all the parts of which once transferred will actually be established as resting, unless it be granted as truly to be moved by participating in the motion of the whole, and hence it has innumerable motions according to the truth of things.

But we see, moreover, from the consequences how absurd this Cartesian doctrine is. And from the first just as he insisted bitterly (that) the earth is not moved because it is not transferred from the vicinity of the contiguous ether; just so will it follow from the same principles that the internal particles of two bodies, when they are not transferred from the immediate vicinity of the co-tangent particles, have no motion properly (so) called, but are moved inasmuch as participating in the motion of the external particles: on the contrary, in respect of which external (parti-

cles) the inner portions would not be moved by a proper motion, because they would not be transferred from the vicinity of the internal parts: and thus in respect of which only the external surfaces of each body are moved by a proper motion, and that the whole internal substance, that is, the whole body is moved through participating in the motion of the external surface. He erred, therefore, in defining motion fundamentally, as he assigned it to bodies what is only suited to surfaces, and he did it so that nothing is able to be the proper motion of any body whatever.

Secondly. What if we look only at Article 25, Part 2? Every body will have not only one but innumerable motions proper to itself, whenever properly and according to the truth of things that of which the whole is properly moved will be said to be moved. And because he defines that motion by means of this body, he understands all that which simultaneously is transferred, and yet himself agrees that another motion is possible from the parts possessing (it) among themselves; suppose the vortex at one with all of the planets or a floating ship at one with the whole sea of which they are part [to which they appertain], or a man in a ship at one with the things which are carried about with himself, or the revolving metal of a watch at one with the particles of which they are made. Now, unless you say that he does not posit the motion of the whole aggregation, motion properly and according to the truth of things suiting the parts, he will be acknowledging that all these motions of the clock's revolutions, the men, the ship, and the vortex actually and philosophically speaking will appertain to the revolving particles.

And whether from these consequences it is besides manifest that nothing absolutely true and proper can be said about the motions on account of anything else, but that all, whether with respect to the contiguous bodies or detached bodies, are the same philosophically, than which we are able to imagine nothing more absurd. Unless it be granted (that) one physical motion is given of every body, and further (that) changing relation and position among other bodies, they are only being designated externally: it will follow that the earth, for example, endeavors to recede from the center of the sun on account of the motion relative to the fixed stars, and endeavors the less to recede on account of the lesser motion relative to Saturn and the ethereal orbit in which it is carried, as well as additionally less relative to Jupiter and the surrounding ether of which its orbit is made up, and again less relative to Mars and its ethereal orbit, and much less relative to the other orbits of ethereal matter which, carrying no planet with them, are properly the annual orbits of earth; truly, relative to its own orbit it does not endeavor, since within it is not moved. All which endeavors and non-endeavors, when they can not absolutely be suitable, it is rather said that one only absolute and natural motion of earth is suited, the example of which is to endeavor to recede from the sun, and that its transferences relative to external bodies are only being designated as external.

Thirdly. It will follow from Cartesian doctrine (that) motion can be generated where no force is being impressed. For example, if God should so effect that the whirling of our vortex would suddenly stand still, no forces being impressed on earth which would simultaneously stand (it) still: Descartes would say that the earth would then be moved in a philosophical sense on account of being transferred from the vicinity of the co-tangent fluid, just as before he had said it to be at rest in the same philosophical sense.

Fourthly. From the same doctrine it will follow also that God himself cannot generate motion in others though he should press the maximum force. For example, if God should ever

press the maximum force so that he might cause the heavenly star at one with the most distant part of creation to revolve around the earth (namely, a diurnal motion); nevertheless, according to Descartes, from this not the heaven but the earth alone would actually be said to be moved (Art. 38, Part 3.) Just as it had been similarly whether he had brought it about that the heaven had been reversed from east to west by means of inborn force, or if by a small force the earth might be reversed in opposing parts. But will anyone judge that a part of the earth is drawn to recede from its center on account of a force in heaven alone being impressed in some manner? Or, is it not more rational, to agree that a force imparted from heaven so produced that to endeavor to recede from the center of the whirling is caused thereby, and by that alone is it properly and absolutely moved; and the force of earth being impressed, that made its parts endeavor to recede from the center of the whirling, is caused thereby, and by that alone is properly and absolutely moved: Even if it is similar in either case to the transference of bodies among themselves. Therefore both physical and absolute motion is being designed by something other than by this transference, for dwelling on this transference is only for designating externals.

Fifthly. It is seen as alien to reason that bodies, apart from physical motion, distances, and positions would change among themselves: But Descartes agrees that the earth and the rest of the planets and fixed stars are properly speaking at rest, and nevertheless change positions among themselves.

Sixthly. And, on the contrary, it is seen to be no less alien to reason, that many bodies preserve among themselves the same positions with respect to which another is physically moved, while the others are at rest. But if God were to stop some planet and make it so that the same position among the fixed stars would be preserved continually, Descartes would not say that with respect to the stars not being moved, the planets then would be moved on account of being transferred from the matter of the vortex.

Seventhly. I ask by what reason any body is said properly to be moved when other bodies from whose vicinity it is transferred are not seen as being at rest, or rather when they cannot be seen as being at rest. For example, in the same manner, our vortex, by reason of the transference of matter following the circumference, can be said to be moved entirely from the vicinity of the other matter surrounding the vortex by revolving, if indeed the matter of the surrounding vortex might not be able to be seen as being at rest, and that not only relative to our vortex, but also insofar as these vortices are not at rest among themselves. Now, if this philosopher assigns being transferred not to the numerable bodily particles of the vortex, but to the generic space (as he says himself) in which these vortices exist, we agree finally, for he would recognize that motion ought to be assigned to space insofar as it is distinguished from body.

Lastly, that the absurdity of his position be exceedingly manifest, I say that from it it will follow (that) nothing is determining the velocity of another moved thing and nothing is defining the line along which it is moved: And much more, that the velocity of a body in motion without impediment can not be said to be uniform, and neither (can there) be a straight line in which it will be carried out. On the contrary, that nothing is able to be moved if, indeed, nothing is able to be so without velocity and determination (destination)].

But that these things may be manifest, the impression having been laid out is that, after any motion having been accomplished, no one can assign the place according to Descartes at which the body was in the beginning of the accomplished motion, or rather he has not said whence it is possible a body will be moved. And the reason is that according to Descartes it is not possible to define and assign the place except from the position of the surrounding bodies, and that after any motion having been accomplished the position of the surrounding bodies remains no more the same as it was before. For example, if the place of the planet Jupiter were where (it was) the year before, then having been accomplished it would be at rest; by what reasoning, I pray, will the philosopher, Descartes, describe it? Not by means of the positions of particles of the fluid matter, if indeed he would have the positions which those particles had the year before, for they will have changed exceedingly. And neither will he describe (it) by means of the positions of the sun and the fixed stars, because of the unequal flux of the subtle matter around the poles of the vortex in the central stars, (Art. 104, Part 3), the swaying, inflation, and absorption of the vortex, and other truer causes, such as the whirling properly around the center of the sun and the stars, the generation of spots, and the trajectory of comets through the heavens, sufficiently changing both the magnitudes and positions of the stars so that by straying they would hardly suffice for the place sought without designating some milestone, and much less that by aid of these it might be possible to determine and thus have described the place accurately, in the same manner as would be postulated for describing the geometry. There are not bodies situated similarly in the world, whose position for length of time are not changed among themselves, and much less those which are not moved in a Cartesian sense, that is, either inasmuch as they are parts of other such transferred bodies. And hence nothing fundamental is given by which the place which was at an earlier time should be able to be designated now in the present, or whence we are able to say such a place has now been more fully situated in the nature of things. Now, when according to Descartes place would be nothing other than the surfaces of surrounding bodies, or the position, as you please, among other more distant bodies: it is impossible in the nature of things that by this doctrine there might exist a longer time than these positions of bodies remain the same from which one would assume individual names [denominations]. And hence, about the place of Jupiter, which it kept the year before, and with equal reason, about the prior place of a moving body anywhere, according to the doctrine of Descartes, it is manifest that not even God himself (standing newly established with things) could accurately and in a geometrical sense describe (it), especially when, on account of the changed positions of bodies, it would no longer exist in the nature of things.

Now, according as any motion having been completed, when the place in which it will have begun—that is, the beginning of the spatial trajectory—cannot be assigned (as) no longer being, nothing keeping the beginning of this spatial trajectory, there cannot be length; hence, when velocity derives from the length of space traversed in the time given, it will follow that there cannot be any velocity of something moving, just as I wished at first to point out. Moreover, what could be said about the beginning of the space traversed ought similarly to be understood about all intermediate places; and thus, when space has neither a beginning nor intermediate parts, it will follow (that) there would not be any space traversed and hence not any motion being determined, the which I wished secondly to indicate. In fact, on the contrary, it will follow (that) Cartesian motion is not motion, since nothing of it is velocity, nothing determinate, and no space anywhere, no distance will be traced. It is accordingly necessary that the determination of places and thus of local motions is represented in some unmoved being of which sort space or

extension alone is as that which is seen (as) distinct from body. And this the Cartesian philosopher would more willingly recognize if he would heed the manner by which Descartes himself had an idea of his extension as distinct from body, which he wished to discriminate from bodily extension [magnitude] by naming it generically. (Article 10, 12, & 18; Part 2). And what he had deduced from the whirling of the vortex, by which the force of the ether was receding from the center, and thus the whole of his mechanical, philosophy, is silently represented by this generic extension.

For the rest when Descartes in Articles 4 & 11, Part 2, *Principiae*, is seen to have demonstrated that body nowise differs from extension, (he is) abstracting no doubt from hardness, color, gravity, cold, heat and the remaining qualities of which body can do without, as if the one thing only remain of it, extension in length, breadth, and depth, which hence alone belongs to its essence. And when this is held along with the many things demonstrated before, and it is alone that to be supposed a cause on account of which faith can be restricted to this opinion; and by that not any doubt about the nature of motion would remain; I will respond to this argument by saying what extension would be, what body, and the manner by which they differ by turns. When, for example, by the distinction of substances into thinking and extended, or rather into being thought and being extended, would be the special foundation of Cartesian philosophy, or what he contends is a notion of mathematical demonstration, its overturning from the side of extension, as if it were built upon the truer foundations of mechanical sciences, I would suppose a small thing by no means.

About extension, then, it is probably expected that it is being defined either as substance or accidents or nothing at all. But by no means nothing, surely, therefore it has some mode of existence proper to itself, by of which it fits neither to substance nor to accident. It is not substance, then, because not absolute in itself, but only the productive [emanative] effect of God, and some affect must subsist in every being; because, then, it would not subsist through the modes of its proper affections which are being denominated substance, that is, through actions inasmuch as they are thinking in the mind and motions in the body. For, although philosophers do not define substance to be the being that is able somehow to act, nevertheless all silently understand this about substance, in the same manner that, for that reason, it is manifest they easily agree (that) extension is substance in the form of body if it can be moved in the manner and bears the fruit of the actions of bodies. And, on the other hand, they by no means agree [concede] (that) body is substance if it can neither be moved nor arouse sensations or any perceptions somewhere in the mind. Moreover, since we can clearly conceive extension alone without existing in some subject, as when any extra-mundane spaces or places are imagined empty of bodies; and we believe (it) to exist anywhere we imagine (that) there is no body, and we cannot believe (that) it will perish along with body if God would somehow destroy some one, it will follow that this does not exist by means of the mode of accident inhering in some subject. And thus it is no accident. And much less could it be called nothing, since what is something more than accident then approaches to the nature of substance. No idea is given of nothing, and neither are there any properties, but we have the clearest idea of all of extension by abstracting, no doubt, the affections and properties of body so that uniformity of space in length, breadth, and depth, and unlimited by division, alone would remain. And moreover there are several of its properties concomitant with this idea, which I shall now enumerate not only as if laying out something (that) is but simultaneously what should be.

1. Space can be distinguished in every way into the parts the common limits of which we are accustomed to call surfaces; and these surfaces can be distinguished in every way into parts, the common limits of which we name lines; and in turn these lines can be distinguished in every way into the parts which we call points. Now [And] these surfaces have no depth, nor lines breadth, nor the point any kind of dimension; unless you would say that spaces are self-limiting mutually to the constant depth of piercing in the surface they penetrate, as being what I have said is the limit of each without common extremity: and thus, too, for lines and points. Further, spaces are everywhere contiguous to spaces, and extension posited everywhere by the side of extension, and thus of co-tangent parts everywhere there are common limits; that is, from this, then, surfaces everywhere being divided by solids, and everywhere lines in which the parts of surfaces are themselves co-tangent, and everywhere points in which the parts of lines are continuously connected. And from this there are all kinds of figures everywhere, spheres everywhere, circles everywhere, ellipses, parabolas, and all the rest, as also every shape and magnitude, although not delineated to sight. For the delineation of some figure is not materially of that figure while space (is) newly produced, but is only its bodily representation so that would now appear to sense which before was insensible in space. Thus, for example, we believe all space in that way to be spherical through which any sphere ever traverses progressively moving in a single moment, albeit of this very sphere there no longer remain any sensible vestiges. Indeed, we believe the space were spherical earlier than the sphere had occupied (it), such that it could itself hold it [the sphere]; and thence, when there are spaces everywhere which can adequately hold any material sphere whatever, it is manifest (that) space is spherical everywhere. And so too about the other figures. In the same manner, within clear water also, we see no material figures, nevertheless several belong (to it) as only some color being inserted by many ways into its differing parts would make to appear. Color, however, if it were inserted, would not constitute the material figures but would only cause (them) to be visible.

2. Space is evenly extended everywhere to infinity. We cannot, for example, imagine anywhere a limit such that simultaneously we are understanding what is given beyond space. And from this all straight lines, paraboloid, hyperboloid, and all cones and cylinders, and the remaining figures of this kind are evenly advanced to infinity, and no one is limited albeit they are intercepted randomly by all kinds of lines and surfaces continuing on crosswise, and among themselves are everywhere constituted the segments of figures. That you might have some true specimen of infinity; suppose some triangle with its base along with one leg [side] at rest and the other leg [side] around the end of its co-tangent base so that it whirls in the plane of the triangle as if the triangle would gradually be opened at the vertex: and meanwhile let the point turn in your mind, where the two legs [sides] come together if in this way they would be produced thus far, and it is manifest that all these points would be situated on the straight line in which the leg [side] at rest lies, and that from this the distance by which the moving leg is continually revolved about it at a perpetually greater reach, the other legs would traverse as far as the parallel and can no longer come together with it anywhere. I ask next how much was the distance of the last point at which the legs came together? Certainly it was more than any (that) can be assigned, or rather not any of the points was last, and hence the straight line in which all these points are found combining is an achievement more than finite. Nor is it what some would say, (that) this is by imagination only and not an achievement of infinity; for if producing a triangle would be an achievement, its legs always being laid out stretched towards some common point (would be) an achievement, if in this way they were produced to where they both come together, and hence the

point where the products come together will always be an achievement of such kind, even if it be supposed to be outside of the world of bodily limits; and thus the line which all these points designate will actually be, however much it be advanced beyond all distance.

Lest someone might now object that we cannot imagine infinite extension, I agree: But meanwhile I contend that we can understand (it). We can imagine a greater extension and also a still greater, but we understand a greater extension than any we can imagine to exist. And from this, incidentally, the faculty of understanding is clearly distinguished from (that) of imagining.

If, however, one would say, moreover, that we do not understand what infinite being would be except by means of negating the limit of the end, and that this is, to that extent, falsifying the conception negatively: I deny it. For the path or limit is the check or the negation of several realities or existences to being by means of limitation, and by which we the less conceive (as) something being restrained being in the limits, wherefore we discover liquid better by positing (it) in itself; that is, for that reason we conceive (it) better positively. And hence by negating all limits the conception turns out altogether positively. End is a name to the extent of the negative sense, and thus far infinity, when it would be the negation of a negation (that is, the end), will be a name entirely positive to the extent of our sense and conception (of it), albeit grammatically it is seen negatively. Add that the finite quantities of several infinite surfaces are known accurately lengthwise by geometry. And thus, I am able to determine positively and exactly the solid quantities of infinite solids now by length, now by width, and to compare them with given finite solids. But this is not the place for that.

What if Descartes, now, would say (that) extension were not infinite but only indefinite: by reason of grammar he is correct. For the name indefinite may not at all be said about that which is an achievement but always with respect to the possibility of the future, only denoting something (that) has not yet been determined as definite. Thus, before God established something by creating the world, (if he at anytime had not established (it)), the quantity of matter, the number of stars, and all the rest would have been indefinite, which therefore will have become defined by the creation of the world. Thus matter is indefinitely divisible, but always either finitely or infinitely divided. (Article 26, Part I; Article 34, Part II) Thus, the indefinite line is that the future length of which would not yet be determined. And so is that space indefinite whose future magnitude would not yet be determined; therefore what is truly an achievement is not to be defined but either does or does not have a limit, and thus is either finite or infinite. It is no objection that he says (it) is indefinite as far as we (know), that is, we are only ignorant of its ends and do not positively know there to be none, (Article 27, Part I): allowing for our ignorance, then, not even God understands there to be none only indefinitely but certainly and positively; on the other hand, because however much we still imagine (it) negatively, we nevertheless positively and certainly understand (that) this transcends all limits. But I see what Descartes apprehended, doubtlessly should space be posited (as) infinite, it would surely become God on account of the perfection of infinity. But by no means, for perfection is not infinity except insofar as it is allotted to perfection. Infinity of intellect, power, happiness, etc., is entirely perfect; infinity of ignorance, impotence, misery, etc., is entirely imperfection; and infinity of extension of whatever kind is the perfection of whatever is extended.

3. The parts of space are immobile. If they be moved, either it is saying that their motion would be transference from the vicinity of other co-tangent parts, just as Descartes has defined the motion of bodies, and that I have sufficiently set forth to be absurd; or it is saying that it would be transference from space to space, that is, out of itself, unless perchance it be said that two spaces coincide everywhere, the mobile and the immobile. The rest of space will best be shown to be immobile by means of duration. In the same manner that, for example, the parts of duration are individuated by means of the order, as if therefore (for the sake of example) the order of yesterday's day could be exchanged with this day, and turn out to be afterwards, the individuation would be lost and there would no longer be yesterday's day but this day: thus, the parts of space are so individuated by means of their positions that if two could exchange positions, however much, they would simultaneously exchange individuation, and each would be numerically converted into the other. On which account, among themselves, the parts of duration and space may be understood to be only the same order and positions themselves which actually are; they have no other principle of individuation beyond these order and positions, which hence are unable to change.

4. Space is being to the extent of being a relation. No being would exist or can exist that is not in some manner attributed to space. God is everywhere, created minds are somewhere and body in the space that it fills; and whatever is neither everywhere nor somewhere, that is not. And hence it follows that space would be the productive [emanative] effect of the existence of being primarily, because by positing being anywhere space is posited. And similarities can be affirmed about duration: obviously both are affections or attributes of being which are denominated according to its quantity of existence in the individual to the extent of the amount [size] of the presence and perseverance in itself. Thus, God's quantity of existence according to duration has been eternal, and according to the space in which he is present, infinite; and the quantity of existence of created things according to duration has been as much as the quantity of duration from the beginning of existence, and according to the size [amount] of the presence as much as also the space in which it is present.

For the rest, then, it cannot be imagined that God corresponds to the image of the extended body and divisible parts: knowing (that) spaces themselves are not a divisible achievement, and beyond any being whatever has a mode proper to itself by which it is present in space. Thus, for example, the relation of duration by length is different to space than to body. For we do not ascribe differences of duration to the differences of parts, but we say all simultaneously endure. The same is the moment of duration at Rome and at London, the same of earth, the heavenly stars, and the universe. And in the same way we understand every single moment of duration thus, by means of universal space, by its own manner, without any part of its concept dispersed: therefore no more does one contradict that mind also could be dispersed conceptually through space in its own manner, without any part.

5. The positions, distances, and local motions of bodies are assigned to parts of space. And that is manifest from the first and fourth surveys of the properties of space, and it will be more manifest if you conceive vacuums to be dispersed among the corpuscles, or you attend to that which I have said first about motion. Let there be subjoined to this, moreover, that there does not appertain to space any force of impeding or advancing or whatever by reason of

the changing motions of bodies. And from this projectile bodies describe straight lines in a uniform motion if not being counteracted by a different one impeding them). But more about this later.

6. Lastly, space is of eternal duration and immutable nature, just as that productive [emanative] effect of being would be eternal and immutable. If ever there were not space, God would then be present nowhere, and hence he created space somewhere after he himself had not been present, or, what is not less discordant with reason, he created his own ubiquity. Again however much we can perhaps imagine nothing to be in space, nevertheless we can not think space not to be; in the same manner we can not think duration not to be, although it would be possible to suppose nothing at all to endure. And that is manifest by means of extra-mundane spaces, which (when we imagine the world to be finite) we can not think not to be, however much they are neither revealed to us by God, nor become known through our senses, nor from spaces within the world to the extent of existing dependently. But it is customarily believed about these spaces that they are nothing. Indeed, they are truly spaces. Space, even if it would be the vacuum of body, nevertheless is not in itself vacuum. And that is something however much spaces are nothing besides. Fifthly, it is acknowledged that spaces are no more spaces where the world exists than where nothing is, unless perhaps you say that God, when he created the world in this space, he simultaneously repeated space in himself. Whatever is thus of more reality in one space than in another, that is in body and not in space; in the same manner it will be more clearly manifest if in this manner that puerile and misleading judgment from childhood be deposed, that extension inheres in bodies only as accidents in the subject, without which it cannot actually exist.

Having described extension, explaining bodily nature remains on the other side. Of this, however, when it would exist not necessarily but by divine will, the explanation will be more uncertain for the reason that it is by no means permitted to know the limits of divine power; obviously matter can have been created by one manner only or, in the opposite case, there are several by which another and thus a different one has been allowed to produce beings most similar to bodies. And however much it seem not at all credible, God can create beings most similar to bodies, of which all their actions bring forth and exhibit phenomena, and nevertheless they would not be bodies in essential and metaphysical constitution: (even) when nevertheless I will no way have a clear and distinct perception in this matter, I would not dare to affirm the contrary, and hence I do not wish positively to say just what would be bodily nature but rather, describing whatever kind of being is in bodies by means of all the similarity of which, we are not able not to perceive (that) the creation is in the power of God, and hence which we can by no means certainly say not to be bodies.

Whenever any man would be conscious in himself that he could move his own body with decision and he also believes that the same power belongs to another man, by which through thinking alone they similarly move their own bodies: the power of moving with choice of anyone at all no wise being denied by God, his thinking is infinitely more powerful and the capacity more prompt. And by an equal reason it is conceded that God by the sole action of thinking or willing could embrace any defined space by certain limits that some bodies not advance [penetrate] into it.

What if he were to exercise this very power, and he should cause that some space should arise in the image of a mountain or whatever body being terminated above the earth impervious to bodies, and thus light and all pressing things would stop or would rebound; it seems impossible that with the aid of our senses (which should be constituted judges in this matter only) we will disclose this space not actually to be body; it were indeed tangible on account of the impenetrability, and visibly opaque and colored on account of the reflection of light, and a blow would resonate for the reason that the neighboring air would be moved by the blow.

Let us suppose, accordingly, various spaces to be disseminated throughout the world, some one of which being defined by certain limits, it would turn out impervious to bodies by divine power, and from the hypothesis it is manifest that this would obstruct, and probably reflect, the motions of bodies, and all the properties of bodily particles would be involved, unless the thing were immobile. Further this impenetrability not always to be conserved in the same part of space but able to be transferred hither and thither according to certain laws, so that nevertheless as if the quantity and form of this impenetrable space were not changed, there would be no property of body which were not suited to this. Let there be formed, the thing tangible, and mobile, and able to be reflected, and to reflect, and by joining things in some way not less constituting a part than any other corpuscle, and I do not see why it can not equally act on our minds and on the contrary be borne, when it were nothing other than the effect of the divine mind within the definite quantity of space being evoked. For it is certain (that) God is able by his own will to move our perceptions, and hence attaches such kind of power to the effect of his will.

In the same manner if several spaces in this manner, both in bodies and in themselves, became impervious, they would sustain all the changes of corpuscles and would exhibit the same phenomena. And thus, in order that if this world were entirely constituted from this mode of being, its condition would hardly seem to be otherwise. And hence these beings either were bodies or most similar to bodies. Now if they were bodies, then bodies we can define to be *determinate entities of extension which God everywhere present affects with certain conditions*: to the extent they are (1) that they be mobile, and for that reason I have not said (they) are the numerical parts of space which are absolutely immobile, but only the definite quantities which are able to be transferred from space to space; (2) that by this mode no two could coincide by any part, whether as if they be impenetrable and hence as if being reflected, counter acting by mutual motions and opposing by certain laws; (3) that they could arouse in created minds various perceptions of the senses and dreams [fancies], and, on the other hand, be moved by itself, not surprising when the description of the origin would be based on this.

For the rest of the explanation, then, it will serve to observe the following:

(1) That it is no aid to the existence of these beings if we represent some unintelligible substance to be given, in which the shape [form] would inhere substantially only by attribution: extension and the achievement of divine will are enough. Extension sustains the place of the substantial subject, in which the shape [form] of body is conserved by means of divine will; and this effect of divine will is the shape [form] or formal reason of body denominating every dimension of space in which body is produced.

(2) These beings were no less real than bodies, nor, I have said, less capable of substance. Whatever, indeed, we believe to belong to the reality of bodies, that is done on account of their

appearances and sensible qualities. And hence these beings, when by this manner they were capable of all qualities, and similarly able to exhibit all these phenomena, we should judge (them) to be no less real by the manner they would exist. They would not the less be substances, if indeed by means of God alone they should equally subsist and endure through accidents.

(3) Between extension and the shape [form] imposed on it (that) such-like bears is the analogy of the sort the Aristotelians posited between prime matter and substantial form; to the extent that they say, doubtlessly, the same matter is capable of every shape [form], and derives the denomination of the number of body from the shapes [form]. Thus, for example, I posit (that) shape [form], however many, can be transferred through any space, and thus (can) denominate body everywhere.

(4) They differ, however, (in) that extension (when it would be and what, and what kind, and how much) has more reality than prime matter, and thus (in) that it can, also be understood, in the same manner and shape [form] as I have assigned to body. If indeed there is some difficulty in the conception, it is not the shapes [forms] which God would insert into space, but in the manner by which he would insert (them). But this would not hold up for a difficulty, if indeed the same thing would occur in the manner by which we move our (bodily) parts, and nevertheless we do not the less believe we can move. If this manner may be known within us, we know by equal reason by what stipulation God, too, can move bodies both from some given place expelling and impeding the limited form, not having been expelled or otherwise, however much on the contrary they can be advanced into [penetrated], that is, so to cause that this space were impenetrable and endued with the shape [form] of body.

(5) Accordingly, I have deduced the description of this bodily nature from the capacity of moving our bodies so that every difficulty in the conception is at length removed by this; and moreover so that (within our inmost conscious) it would be manifest (that) God might have created the world by nothing other than willing the action, in the same manner both we by only willing the action move our own bodies; and above and beyond that the analogy being set forth between our faculties and the Divine faculties is the greater than thus far observed by the philosopher. In the imagining of God creating is our being witness to the holy page. And his image shines more completely within us if by the manner he has shadowed the power of creating and thus the rest of his attributes (it) is equal to our conceded capacities, nor is it prejudicial that we are creatures ourselves, and thus far the evidence of his attributes could not be similarly conceded in us. For although by this reason the power of creating minds is not delineated in any capacity of the created mind, nevertheless the creating mind (when it would be the image of God) is of much greater nature than the body that perhaps more prominently contains (it) within itself.<sup>1</sup> But more overt by moving bodies we do not create something nor are we able to create but we only shadow forth the power of creating. We are not able, for example, so to cause that some spaces would be impervious to bodies, but we only move bodies, and this not of any whatever but only our own to which we are united by the divine constitution and not by our will, nor by any manner whatever but according to whatever laws which God has imposed on us. If, however, someone should prefer (that) I call this our power finite and the lowest degree of power which God the creator has established this would no more derogate from divine power than by his understanding he would derogate the understanding that also befits us to a finite degree, (especially when it is not by our own and independent power but by law imposed by God on us that we

move our bodies). In fact, if someone believe it is possible that God produced some understanding creature so far perfect as by the aid of divine concurrence he could again produce creatures of lesser order, that, therefore, would not derogate from divine power, nor would he be positing the infinitely greater claim, by which creatures may be elicited not only immediately but mediately from other creatures. And thus anyone would probably prefer to posit a soul of the world being created by God, to be (that) on which he imposes this law, that defined spaces would describe bodies by properties, than to believe this service to be immediately furnished by God. And for that reason the world is not to be called the creature of this soul but of God alone, who might create by establishing a soul of such nature that the world would arise necessarily. But I do not see why God himself does not immediately inform space with bodies; provided that, by formal reason we distinguish bodies from the achievement of divine will. It has indeed been contradicted that it would itself be the achievement of willing, or something other than the effect only which that achievement produces in space. In whatever the effect differs no less from that achievement than Cartesian space, or the substance of bodies according to the vulgar conception, if by this mode it is created, we suppose that existence is to be derived from the will or to be the being of divine reason.

Lastly, having described the greatest use of the idea of bodies has illuminated that it has most clearly involved, and should confirm and explain, particular truths of metaphysics. For example, we are not able to posit bodies by this manner, except that we simultaneously posit God to exist, and he would have created bodies in empty space *ex nihilo*, and it is the being distinguished from minds, but able nevertheless to be united with mind. Say, pray, which opinion of those now being published, therefore, might elucidate some one of these truths or rather is not opposed to everyone and restores the perplexity. If we say with Descartes (that) extension is body, do we not rather manifestly spread the way to atheism, for then that extension is not being created but was from eternity, whereupon we have an absolute idea of it without any relation to God, and thus we are able to conceive existence for the time being as if at that time we would suppose God not to be. And no distinction of mind from body, according to this philosophy, is understandable, lest simultaneously we say that mind is by no means extension, and thus is substantially present in no extension, or is no place; and so too if we say it is not that by means of which it is seen; however, I have plainly restored its minimum understandable union with body, not saying (it is) impossible. Moreover, if the distinction of substances into thinking and extended is lawful and perfect, then God does not eminently contain, and hence he cannot create, extension within himself; but God and extension are two substances severally called absolutely complete and singular. On the other hand, however, if extension is eminently contained within God or within all thinking being, certainly the idea of extension will be eminently contained in the idea of thinking, and hence the distinction of the ideas will be not *insofar as but as if* {emphasis added} both could be suited to the same thing of created substance, that is body to thinking or the thinking thing extended. Now, if the vulgar idea, or rather non-idea of body is embraced, obviously that bears in body some non-understandable reality, as they say the substance is (that) in which their qualities inhere: This (beyond that which is not understandable) as well as the Cartesian arguments is accompanied by the same inconveniences. For when it cannot be understood, it is impossible that his distinction from the substance of the mind be understood. Nor indeed has assuming the separation from the shape [form] of substance or the attributes of substance sufficed; for if exposed substances have no essential difference, the same substantial forms or attributes are able to be altered to suit and (as if by change, even if it not be simultane-

ous) to bring about mind and body. And thus if we do not understand this difference of the exposed attributes of substances, neither can we knowingly affirm that mind and body differ. Or if they do differ, we are not able to discern any foundation of union. Moreover, in this the proved substances of bodies without qualities and shapes [forms] are assigned somewhat less reality in words, but not less in concept, than the substances of God abstracted from his attributes. Both are similarly conceived tried [proved] nakedly, or rather are not conceived except in common with whatever of intelligible reality they do not confuse with apprehension. And from this it is not astonishing that atheists are born [generated] ascribing to bodily substances that which ought only to be of the divine. Fifthly, in looking about, nothing else of the atheists has quite appeared, as a result of which this notion of bodies, as is possessing complete reality absolutely and independently in themselves, such as for the most parts we all are wont from childhood, lest I err through neglect, to conceive mentally, such as that we verbally say to be created and dependent. I believe it would have been both by reason and this prejudice, that the name of substance is singularly attributed of God and of created beings in the Schools, and that the idea being formulated of body would continue with the philosopher and be indulged vaguely, since he endeavors at the same time to formulate an independent idea of things dependent on God. For certainly whatever can not be independently of God, cannot truly be understood independent of the idea of God. God would no less sustain himself by created beings than they are sustained by accidents, to the same degree that created substance, whether you observe the degree of dependence or the degree of reality, is an intermediate nature between God and accidents. And hence its idea no less involves the conception of God than the idea of accident (involves) the conception of created substance. Thus no reality other than the derived and incomplete ought to be fulfilled in itself. Letting go of the prejudice is accordingly premised and it is rather the imputing the substantial reality of this mode of attributes, which are real by themselves, both understandable and not needing a subject in which they inhere, than a subject in which we can not conceive that as dependent nor formulate any idea of it. And we will hardly do this with grudging if (beyond the idea of body exposed above) we acknowledge in our souls (that) we can conceive space without any subject existing, provided we think the vacuum. And hence something fits in this reality of substance. But if, moreover, the mobility of the part (as Descartes supposed) were involved in this idea, no one would soundly nor easily concede substance to be body. In the same manner, if we have the idea of an attribute or of this power by which God, by action of willing alone is able to create being; perhaps we will conceive this attribute only in itself, without any substantial subject subsisting and involving the rest of its attributes. But meanwhile, while we are not able to formulate the idea not only of this attribute but also of the proper power by which we move our own bodies; it would be to chance to say what thing might be the foundation of mind substantially.

Thus much about the nature of body: in explaining which I believe myself sufficiently to have surpassed what corresponds most clearly to the kind I have revealed, whereby creation is in the power of God, and out of which creation if this very world should not be constituted, at least some other similar (world) by this can be constituted. And there would be differences in matters to the extent of the properties and nature, but only in the method by which God could create one thing as well as another: surely the distinction of body from extension is hereby sufficiently illuminated. For obviously extension would be eternal, infinite, uncreated, uniform throughout, to no extent at all mobile, nor able to induce motions in bodies or thinking in minds; body itself truly has an opposed manner in all this, save if it had not pleased God to have created always and

everywhere. For I would not dare to deny in God this very power. [And if anyone perceive otherwise, he should say where prime matter has been able to be created and whence the power of creating, therefore, has been conceded in God. Or if there were no beginning of this power, but it has kept the same from eternity as it keeps now, then he has been able to have created from eternity. For it is the same thing to say that there never was powerlessness for creating in God, or that he always had the power of creating and has always been able to be created. In the same manner is he assigned the space in which matter at the beginning has not been able to be created, or it should be conceded (that) God, then, has been able to have created everywhere.]<sup>ii</sup>

As for the rest of the Cartesian argument I now respond more strictly: we take from body (just as he bids) gravity, duration, and all sensible qualities, so that nothing finally would remain except what belongs to its essence. Will, accordingly, extension only remain? Not at all. For we reject additionally that capacity or power by which the perceptions of thinking things move. For when the distinction is only between the ideas of thinking and extension so that something would not be manifest to be the foundation of the connection or relation unless that be caused by divine power; the capacities of bodies can be rejected with this reserved extension, but it would not be rejected with the reserved bodily nature. Obviously, the changes which can be induced in bodies by natural causes are only accidental and not denoting the substance actually to be changed. But if anything could induce the change which transcends natural causes, it is more than accidentally and has radically attained the substance. According to the sense of the demonstration those only are being rejected of which body, by force of nature, can be void and deprived. But no one would object that bodies which are not united to minds cannot immediately move their perceptions. And hence when bodies are given united to minds by nothing, it will follow (that) this power is not among their essentials. The observation is that this does not act by actual union but only by the capacity of bodies by which they are capable of this union by force of nature. As by whatever capacity belongs to all bodies, it is manifest from it that the parts of the brain, especially the more subtle by which the mind is united, are in continual flux, the new ones succeeding to those flying off. And it is not lesser to take (off) this, whether regarding the divine achievement or bodily nature, than to take (off) the other capacity by which bodies in themselves are able alternately to transfer mutual actions, that is, than to force body back into empty space.

When, however water would resist the motions of solid trajectories less than quicksilver, and air much less than water, and ethereal space still less than air, we further reject every force of impeding the motion of trajectories, and [then] reasonably we will reject bodily nature. In the same manner if the subtle matter be deprived of every force of impeding the motion of globules, I would no longer believe there to be subtle matter but a dispersed vacuum. Accordingly, if in this way the aerial or ethereal space would be by its own manner so that the projectiles of comets or whatever bodies you wish would yield to motions without any resistance, I would believe (it) to be utterly empty. For it is impossible that bodily fluids would not resist the motions of trajectories, namely, if it be not disposed to the motion in accord with the velocity of its motion (Part 2, Epistle 96, To Mersenne),<sup>iii</sup> just as I have assumed.

Also all this force can be taken to be manifest throughout space, if by a manner space and body alternately differ; and hence (that) I have taken to be possible is not to be denied sooner than they are proved not to differ, lest we equate (them), it being allowed by *petitio principii*.

But (that) not any doubt would remain, it derives from the observation of the aforesaid that empty spaces are given in the nature of things. For if fluid were entirely bodily without any vacuous pores, that, by means of dividing the subtlest parts however, were equally dense and thus any other fluid whatever, and it would not yield less to the inertial motion of trajectories, on the contrary, much more, if the projectile in this way were porous; for the reason that it would penetrate its inmost pores, and not by way of the whole external surface but also it would come together and be by the impeding of all the internal parts on the surfaces. But with the ether on the other hand, forasmuch as there is a little resistance collected so that in resisting quicksilver, it is seen to be more than ten or a hundred thousand less in force: surely much the greatest part of space ought to be kept for vacuum, being dispersed among the ethereal corpuscles. That same thing, moreover, would be granted by collecting the differing gravity of these fluids, as is that of their densities or that the contents (of their) material quantities show in space, here the descending of the heavier, there the swaying of the pendulum. But it is not now the place for explaining this.

Let it be seen accordingly how fallacious and false Descartes's very argument is, (as) if someone rejecting the accidents of bodies not only would preserve extension as if that could be supposed, but also the capacities which are able to move, now perceptions of the mind, now other bodies. For if we reject, moreover, these very capacities and every power of moving so that the precise conception of uniform space would remain, is there any vortex, is there any world Descartes will fabricate out of this extension? No one would reasonably invoke (it), lest God first, who alone would be able (replacing these capacities or bodily nature in the proportion he had before arranged) newly to beget bodies in this space. Accordingly, in virtue of the foregoing, I have assigned bodily nature to consist in the then enumerated capacities.

And so at last, when spaces are not themselves bodies but only the places to which they belong and in which they are moved, which I have defined sufficiently by local motion, I esteem (them) to be established. I do not see what more could be required in this matter, unless perhaps that for them this does not satisfy, I would teach them that by means of space, the parts of which are filled with bodies, I have defined places understanding the Cartesian generic space in which spaces considered particularly, or Cartesian bodies, are moved, and surely they will hardly keep refuting that in our definitions.

Now that I have digressed enough, let us return to the proposition.

[What follows is the translation of A. Rupert Hall and Marie Boas Hall:]

Definition 5. Force is the causal principle of motion and rest. And it is either an external one that generates or destroys or otherwise changes impressed motion in some body; or it is an internal principle by which existing motion or rest is conserved in a body, and by which any being endeavors to continue in its state and opposes resistance.

Definition 6. Conatus [endeavor] is resisted force, or force in so far as it is resisted.

Definition 7. Impetus is force in so far as it is impressed on a thing.

Definition 8. Inertia is force within a body, lest its state should be easily changed by an external exciting force.

Definition 9. Pressure is the endeavor of contiguous parts to penetrate into each others' dimension. For if they could penetrate the pressure would cease. And pressure is only between contiguous parts, which in turn press upon others contiguous to them, until the pressure is transmitted to the most remote parts of any body, whether hard, soft or fluid. And upon this action is based the communication of motion by means of a point or surface of contact.

Definition 10. Gravity is a force in a body impelling it to descend. Here, however, by descent is not only meant a motion towards the center of the Earth but also towards any point or region, or even from any point. In this way if the conatus of the anther whirling about the Sun to recede from its centre be taken for gravity, the aether in receding from the Sun could be said to descend. And so by analogy, the plane is called horizontal that is directly opposed to the direction of gravity or conatus.

Moreover, the quantity of these powers, namely motion, force, conatus, impetus, inertia, pressure and gravity may be reckoned in a double way: that is, according to either intension or extension.

Definition 11. The intension of any of the above-mentioned powers is the degree of its quality.

Definition 12. Its extension is the amount of space or time in which it operates.

Definition 13. Its absolute quantity is the product of its intension and its extension. So, if the quantity of intension is 2, and the quantity of extension 3, multiply the two together and you will have the absolute quantity 6.

Moreover, it will help to illustrate these definitions from individual powers. And thus motion is either more intense or more remiss, as the space traversed in the same time is greater or less, for which reason a body is usually said to move more swiftly or more slowly. Again, motion is more or less in extension as the body moved is greater or less, or as it is acting in a larger or smaller body. And the absolute quantity of motion is composed of both the velocity and the magnitude of the moving body. So force, conatus, impetus or inertia are more intense as they are greater in the same or an equivalent body: they have more extension when the body is larger, their absolute quantity arises from both. So the intension pressure is proportional to the increase of pressure upon the surface-area; its extension proportional to the surface pressed. And the absolute quantity results from the intension of the pressure and the area of the surface pressed. So, lastly, the intension of gravity is proportional to the specific gravity of the body; its extension is proportional to the size of the heavy body, and absolutely speaking the quantity of gravity is the product of the specific gravity and mass of the gravitating body. And whoever fails to distinguish these clearly, necessarily falls into many errors concerning the mechanical sciences.

In addition the quantity of these powers may sometimes be reckoned from the period of duration; for which reason there will be an absolute quantity which will be the product of inten-

sion, extension and duration. In this way if a body [of size] 2 is moved with a velocity 3 for a time 4 the whole motion will be  $2 \times (3 \times 4$  or 12) [sic].

Definition 14. Velocity is the intension of motion, slowness is remission.

Definition 15. Bodies are denser when their inertia is more intense, and rarer when it is more remiss.

The rest of the above-mentioned powers have no names.

It is however to be noted that if, with Descartes or Epicurus, we suppose rarefaction and condensation to be accomplished in the manner of relaxed or compressed sponges, that is, by the dilation and contraction of pores which are either filled with some most subtle matter or empty of matter, then we ought to estimate the size of the whole body from the quantity of both its parts and its pores in Definition 15; so that one may consider inertia to be remitted by the increase of the pores and intensified by their diminution, as though the pores, which offer no inertial resistance to change, and whose mixtures with the truly corporeal parts give rise to all the various degrees of inertia, bear some ratio to the parts.

But in order that you may conceive of this composite body as a uniform one, imagine its parts to be infinitely divided and dispersed everywhere throughout the pores, so that in the whole composite body there is not the least particle of extension without an absolutely perfect mixture of parts and pores thus infinitely divided. Certainly it suits mathematicians to contemplate things in the light of such reasoning, or if you prefer in the Peripatetic manner; but in physics things seem otherwise.

Definition 16. An elastic body is one that can be condensed by pressure or compressed within narrower limits; and a non-elastic body is one that cannot be condensed by that force.

Definition 17. A hard body is one whose parts do not yield to pressure.

Definition 18. A fluid body is one whose parts yield to an overwhelming pressure.<sup>iv</sup> Moreover, the pressures by which the fluid is driven in any direction whatsoever (whether these are exerted on the external surface alone, or on the internal parts by the action of gravity or any other cause), are said to be balanced when the fluid rests in equilibrium. This is asserted on the assumption that the pressure is exerted in some one direction and not towards all at once.

Definition 19. The limits defining the surface of the body (such as wood or glass) containing the fluid, or defining the surface of the external part of the same fluid containing some internal part, constitute the vessel of fluid [*vas fluidi*].

In these definitions, however, I refer only to absolutely hard or fluid bodies, for one cannot ratiocinate mathematically concerning ones partially so, on account of the innumerable circumstances affecting the figures, motions and contexture of the least particles. Thus I imagine that a fluid does not consist of hard particles, but that it is of such a kind that it has no small portion or particle which is not likewise fluid. And moreover, since the physical cause of fluidity is

not to be examined here, I define the parts not as being in motion among themselves, but only as capable of motion, that is, as being everywhere so divided one from another that, although they may be supposed to be in contact and at rest with respect to one another, yet they do not cohere as though stuck together, but can be moved separately by any impressed force and can change the state of rest as easily as the state of motion if they move relatively. Indeed, I suppose that the parts of hard bodies do not merely touch each other and remain at relative rest, but that they do besides so strongly and firmly cohere, and are so bound together, as it were by glue, that no one of them can be moved without all the rest being drawn along with it; or rather that a hard body is not made up of conglomerate parts but is a single undivided and uniform body which preserves its shape most resolutely, whereas a fluid body is uniformly divided at all points.

And thus I accommodated these definitions not to physical things but to mathematical reasoning, after the manner of the Geometers who do not accommodate their definitions of figures to the irregularities of physical bodies. And just as the dimensions of physical bodies are best determined from their geometry (as the measurement of a field from plane geometry, although a field is not a true plane; and the measurement of the Earth from the doctrine of the sphere even though the Earth is not precisely spherical) so the properties of physical fluids and solids are best known from this mathematical doctrine, even though they are not perhaps absolutely nor uniformly fluid or solid as I have defined them here.

### **Propositions on Non-Elastic Fluids**

#### **Axioms**

1. From like postulates like consequences ensure.
2. Bodies in contact press each other equally.

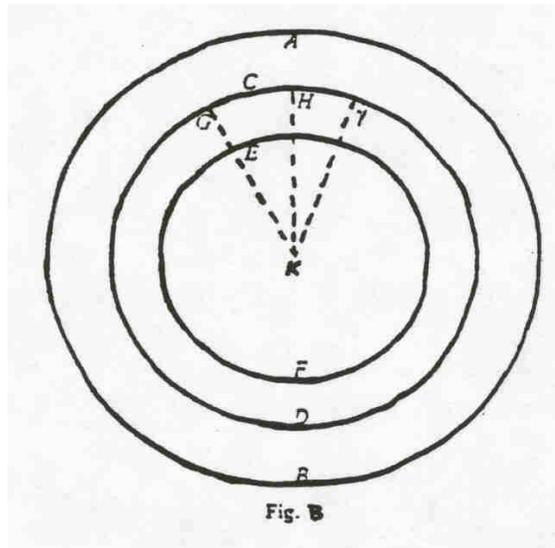
Proposition 1. All the parts of a non-gravitating fluid, compressed with the same intention in all directions, press each other equally or with equal intension.

Proposition 2. And compression does not cause a relative motion of the parts.

#### **Demonstration of Both**

Let us first suppose that the fluid is contained and uniformly compressed by the spherical boundary AB whose center is K [Fig. B]. Any small portion of it CGEH is bounded by the two spherical surfaces CD and EF described about the same center K and by the conical surface GKH whose vertex is at K. And it is manifest that CGEH cannot in any way approach the center K because all the matter between the spherical surfaces CD and EF would everywhere approach the same center for the same reason,<sup>a1</sup> and so would penetrate the volume of the fluid contained within the sphere EF.<sup>b1</sup> Nor can CGEH recede in any direction towards the circumference AB because all that shell of fluid between DC and EF would similarly recede for the same reason<sup>a1</sup> and so would penetrate the volume of fluid between the spherical surfaces AB and CD.<sup>b1</sup> Nor can it be squeezed out sideways, say towards, H, since if we imagine another little section Hy, terminated in every direction by the same spherical surfaces and a similar conical surface and contiguous to GH and H, this section Hy may for the same reason be squeezed out towards H,<sup>a1</sup> and so effect a penetration of volume by the mutual approach of contiguous parts.<sup>b1</sup> And so it is

that no portion of fluid CGEH can exceed its limits because of pressure. And hence all the parts remain in equilibrium. Which is what I wished to demonstrate first.

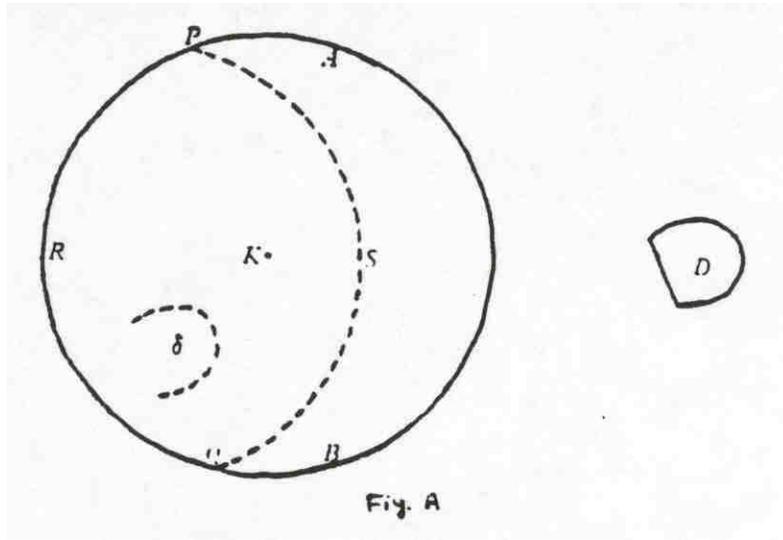


I say also that all parts press each other equally, and with the same intension of pressure that the external surface is pressed. To show this, imagine that PSQR is a part of the said fluid AB contained by similar spherical segments PRQ and PSQ, and that its compression upon the internal surface PSQ is as great as that upon the external surface PRQ [Fig. A ]. For I have already shown that this part of the fluid remains in equilibrium, and so the effects of the pressures acting on both of its surfaces are equal, and hence the pressures are equal.<sup>c1 d1</sup>

And thus since spherical surfaces such as PSQ can be described anywhere in the fluid AB, and can touch any other given surfaces in any points whatever, it follows that the extension of the pressure of the parts along the surfaces, wherever placed, is as great as the pressure on the external surface of the fluid. Which is the second point I wished to demonstrate.

Moreover, as the force of this argument is based on the equality of the surfaces PRQ and PSQ, lest it should seem that there is some disparity, in that one is within the fluid and the other is a segment of the external surface, it will help to imagine that the whole sphere AB is a part of an indefinitely larger volume of fluid, in which it is contained as within a vessel, and is everywhere compressed just as its part PRQS is pressed upon the surface PSQ by another part

PABQS. For the method by which the sphere AB is compressed is of no significance, so long as its compression is supposed to be equal everywhere.



Now that these things have been demonstrated for a fluid sphere, I say lastly that all the parts of the fluid D (bounded in any manner at all, and compressed with the same intension in all directions) press each other equally and are not made to move relatively by the compression. For let AB be an indefinitely greater fluid sphere compressed with the same degree of intension; and let be some part of it equal and similar to D. From what has already been demonstrated it follows that this part is compressed with an equal intension in all directions and that the intension of the pressure is the same as that of the sphere AB, that is (by hypothesis) as that which compressed the fluid D. Thus the compression of the similar and equal fluids, D and \*, is equal; and hence the effects will be equal.<sup>a2</sup> But all the parts of the sphere AB<sup>b2</sup> and so of the fluid contained in it, press each other equally, and the pressure does not cause a relative motion of the parts. For which reason the same is true of the fluid D.<sup>a2</sup> Q.E.D.

Corollary 1. The internal parts of a fluid press each other with the same intension as that by which the fluid is pressed on its external surface.

Corollary 2. If the intension of the pressure is not everywhere the same, the fluid does not remain in equilibrium. For since it stays in equilibrium because the pressure is everywhere uniform, if the pressure is anywhere increased, it will predominate there and cause the fluid to recede from that region.<sup>c2</sup>

Corollary 3. If no motion is caused in a fluid by pressure, the intension of the pressure is everywhere the same. For if it is not the same, motion will be caused by the predominant pressure.<sup>d2</sup>

Corollary 4. A fluid presses on whatever bounds it with the same intension as the fluid is pressed by whatever bounds it, and vice versa. Since the parts of a fluid are certainly the bounds of contiguous parts and press each other with an equal intension, conceive the aforesaid fluid to be part of a greater fluid, or similar and equal to such a part, and similarly compressed, and the assertion will be evident.<sup>a2</sup>

Corollary 5. A fluid everywhere presses all its bounds if they are capable of withstanding the pressure applied, with that intension with which it is itself pressed in any place. For otherwise it would not be pressed everywhere with the same intension.<sup>e2</sup> On which assumption it yields to the more intense pressure.<sup>d2</sup> And so it will either be condensed<sup>f2</sup>, or it will break through the bounds where the pressure is less.<sup>f2</sup>

Scholium. I have proposed all this about fluids, not as contained in hard and rigid vessels, but within soft and quite flexible bounds (say within the internal surface of a homogeneous exterior fluid), so that I might more clearly show that their equilibrium is caused only by an equal degree of pressure in all directions. But once a fluid is put into equilibrium by an equal pressure, it is all one whether you imagine it to be contained within rigid or yielding bounds.

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\* This is a translation of the manuscript based on the text authenticated by A. Rupert Hall and Marie Boas Hall. The conclusion of the essay, from “Def. 5” is offered in the Halls’ translation, which offers no special difficulty. For the rest, this rather ungracious, rough and ready translation pretends to offer the advantage of important literalness and consistency in important passages, at least for as much as it is correct. The serious reader is urged to consult Newton’s Latin, in *Unpublished Scientific Papers of Isaac Newton*, ed. by A. Rupert Hall and Marie Boas Hall, (Cambridge: Cambridge University Press, 1962), pp.90-121. As the editors point out, the argument here is heavily echoed in the General Scholium of the *Principia*, as well as elsewhere. For reasons that are clear, I find that the grounds of Newton’s opposition to Descartes are much more comprehensive here, and more useful by way of the general analysis of the relation of the mechanical sciences to the human sciences. Bracketed entries indicate alternative, usually more literal translations; parentheses are either Newton’s own or indicate necessary grammatical structures which complete thoughts in English but are not actually required in Latin.

<sup>i</sup> “...that it more prominently contains within itself.” –Hall & Hall

<sup>ii</sup> “The last four sentences of this paragraph were added in a marginal note.” – Hall & Hall

<sup>iii</sup> “The not in the bracket is written in the margin of the ms., with an asterisk to show its place in the text.”—Hall & Hall

<sup>iv</sup> Newton originally wrote all whose parts are mobile among themselves, but crossed out these words.

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- a<sup>1</sup> Axiom 1.
  - b<sup>1</sup> Contrary to the definition.
  - a<sup>1</sup> Axiom 1.
  - b<sup>1</sup> Contrary to the definition.
  - a<sup>1</sup> Axiom 1.
  - b<sup>1</sup> Contrary to the definition.
  - c<sup>1</sup> Axiom.
  - d<sup>1</sup> Definition.
  - a<sup>2</sup> Axiom.
  - b<sup>2</sup> According to what has already been demonstrated.
  - a<sup>2</sup> Axiom.
  - c<sup>2</sup> By definition.
  - d<sup>2</sup> Corollary 2.
  - a<sup>2</sup> Axiom.
  - e<sup>2</sup> Corollary 4.
  - d<sup>2</sup> Corollary 2.
  - f<sup>2</sup> Contrary to the hypothesis.